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## THE DEVELOPMENT OF THE TURBULENT BOUNDARY LAYER ON STEEP SLOPES

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ENGINEERING MECHANICS DIVISION
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PAPERS

# TURBULENT BOUNDARY LAYER ON STEEP SLOPES

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## Synopsis

The development of the turbulent boundary layer in a fluid flowing down steep slopes controls not only the loss of head, but also the air entrainment. Prediction of the rate of growth of the layer is essential to proper spillway design. This paper describes a laboratory investigation of the variation in velocity distribution on smooth and rough channel beds for varying slopes and rates of discharge. The boundary-layer thickness, the coefficient of local resistance, and the parametric shape of the velocity profile are analyzed as functions of the Reynolds number and the relative roughness. Although these functions are shown to be in accordance with the general boundary-layer theory, they are essentially independent of both slope and discharge. A practical method is proposed for computing the limit of boundary-layer development on concrete spillways.

## Introduction

*Notation.*—The letter symbols introduced in this paper are defined where they first appear, in the text, or by illustration, and are assembled alphabetically in the Appendix for convenience of reference.

The boundary layer which develops on the steep slope of a spillway has important effects on the entire behavior of the fluid flow. The most important effect is the fact that the free surface of the flow may be made turbulent if the boundary layer attains a thickness approaching the thickness of the flow. Since a turbulent free surface is a prerequisite to air entrainment, and since the boundary layer is the primary source of that turbulence, the significance of the layer's development on steep slopes lies in its connection with the air entrainment process and with the problems associated with the resulting bulking of flow.

The boundary layer influences the depth, the velocity, and the turbulence

of the flow, and through its precipitation of the air entrainment process, also affects the density of the fluid. Without precise knowledge of these aspects of the flow and this property of the fluid, the design of structures associated with spillway floors becomes necessarily approximate, whereas an exact design would often be more economical.

## Background of the Problem

*Theory of the Boundary Layer Applied to Steep Slopes.*—The assumption of a non-viscous fluid yields a solution to the well-known engineering problem of flow down steep slopes. Through application of the requirements of irrotationality and of the principles of conservation of energy and matter, together with the condition that the pressure be constant along the free surface, the velocity, the pressure, and the location of the free surface at any station along the slope can be determined. The curvilinear geometry of the boundary makes the analysis tedious, but offers no fundamental difficulty.

When viscosity is included in the problem, however, the expedient of separating the flow into two domains is used, as suggested by L. Prandtl in his original statement of the boundary layer concept.[2] Mr. Prandtl considered that in many flow situations (particularly those involving fluids of low viscosity, of which water is a good example), the viscous effects are confined within a relatively narrow zone adjacent to the boundary; the greater part of the flow therefore behaves as if the fluid were non-viscous. Mr. Prandtl then proposed that these two domains be treated separately. The domain of large viscous effects is called the boundary layer; the domain of negligible viscous effects is termed the zone of near-potential flow, indicating the flow that is only slightly different from the potential flow of the non-viscous fluid. On a spillway, both domains exist contiguously—the near-potential flow being dominant at the upstream end of the spillway, and the boundary layer increasing in thickness in the direction of flow.

[2] "Über Flüssigkeiten bei sehr kleiner Reibung," by L. Prandtl, *Verhandlung*, Third International Mathematics Congress, Heidelberg, Germany, 1904.

The flow in the near-potential domain remains essentially irrotational and free of shear forces. Flow in this domain is handled as if it were flow of a non-viscous fluid, except for its displacement in the direction away from the boundary. In a spillway problem, such displacement is never large enough to produce a significant difference between the magnitude of the actual velocity in the near-potential domain and that computed at the same station on the basis of irrotational flow. The displacement is significant in determining the location of the free surface, however.

The flow within the boundary layer is complicated by the presence of rotational flow and the internal shear, which reaches a maximum at the boundary. The nature of these two complications is understood only in the case of laminar flow, in which the viscous-shear relationship applies. The development of laminar boundary layers may be handled without resort to experimental measurement. Unfortunately, the transition from the laminar to the turbulent boundary layer usually occurs far upstream from the zone of interest, so that the laminar boundary layer constitutes no significant part of the spillway problem.

The development of the turbulent boundary layer is governed by parameters which have customarily been determined by experiment. Similarly, the proposed solution depends on laboratory observations of the desired parameters. These parameters are then analyzed with respect to experience with turbulent flows. This empirical approach is the only one available in view of the lack of knowledge of the principles which govern turbulent flow.

In addition to locating both the point at which the free surface of the flow first becomes turbulent (because of the proximity of the turbulent boundary layer) and the free surface upstream from it, the aim of this study was to delineate the effect of continuous positive acceleration on the standard boundary-layer parameters. This latter goal culminates in a series of functional relationships presented graphically in a subsequent section. For better understanding of these relationships and their significance, a brief review of the analytical background of this study is necessary.

*Definitions of Boundary-Layer Length Parameters.*—In the case of turbulent boundary layers, the analysis is based on experimental evidence, which is interpreted according to a commonly accepted set of definitions. The nominal thickness of the boundary layer is designated by the symbol $\delta$. This thickness is the normal distance from the boundary at which the magnitude of the velocity is approximately 99% of the value predicted by the theory of irrotational flow. A graphical presentation of this relationship is shown in Fig. 1.

In addition to the nominal thickness, two parameters of other lengths are commonly used in the study of boundary layers. One of these is the displacement thickness, $\delta_d$, which is the distance from the boundary that the potential flow has been displaced by the presence of the boundary layer. The other parameter is the momentum thickness, $\delta_m$, which is a measure of the defect of momentum flux within the boundary layer. For two-dimensional flow, these two lengths are defined as

$$\delta_d = \int_0^\delta \left(1 - \frac{u}{U}\right) dy \qquad (1)$$

and

$$\delta_m = \int_0^\delta \left[\frac{u}{U} - \left(\frac{u}{U}\right)^2\right] dy \qquad (2)$$

in which $u$ is the velocity at a point within the boundary layer, $U$ denotes the velocity of the potential flow, and $y$ is the normal distance from the boundary.

*The von Kármán Momentum Equation.*—Using the two length parameters $\delta_d$ and $\delta_m$, T. von Kármán, M. ASCE, formulated the following expression for the boundary shear in accelerating flows:[3]

[3] "Laminare und turbulente Reibung," by T. von Kármán, *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 1, 1921, p. 235.

$$\frac{2\,\tau_0}{\rho\,U^2} = c_f = 2\frac{d\delta_m}{dx} + (\lambda + 2)\frac{\delta_m}{U^2}\frac{dU^2}{dx} \qquad (3)$$

in which $c_f$ is the local coefficient of boundary shear, $\tau_0$ represents the intensity of boundary shear, $\rho$ is the mass density of the fluid, $x$ denotes the distance in

the direction of flow, and $\lambda$ is a shape parameter of the velocity profile defined as $\delta_d/\delta_m$.

If the three functions that relate $U$, $c_f$, and $\Lambda$ to $x$ are known, then this expression can be numerically integrated to obtain $\delta_m$ as an explicit function of $x$. Of these three functions the first is given in the usual case, and the other two are often assumed according to the concepts of the boundary-layer behavior indicated by related experiments.

*The von Kármán-Prandtl Velocity-Distribution Equations.*—Many of the ideas concerning boundary layers find their basis in an abundance of experimental data concerning flow in pipes. Among them are the von Kármán-Prandtl velocity-distribution equations for turbulent flow near smooth and rough boundaries. The expressions given by Hunter Rouse, M. ASCE, are

$$\frac{u}{\sqrt{\frac{\tau_0}{\rho}}} = 5.75 \log_{10}\left(\frac{\sqrt{\frac{\tau_0}{\rho}}}{\nu} y\right) + 5.5 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

for smooth boundaries, and

$$\frac{u}{\sqrt{\frac{\tau_0}{\rho}}} = 5.75 \log_{10} \frac{y}{K} + 8.5 \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

for rough boundaries.[4] These expressions were derived by Mr. von Kármán

[4] "Elementary Mechanics of Fluids," by Hunter Rouse, John Wiley and Sons, Inc., New York, N. Y., 1946.

by application of the Prandtl mixing-length theory, assumption of a constant shear stress throughout the region, and neglect of the laminar shear. The experimental data of J. Nikuradse[5] and others confirm the derivation.

[5] "Strömunggesetze in rauhen Rohren," by J. Nikuradse, *Forschungsheft*, 361, Verein Deutsches Ingenieurs, Berlin, Germany, 1933.

*Related Investigations.*—Boundary-layer developments in uniform flow on flat plates have been both synthesized by a mathematical conversion of pipe-flow principles and measured in the laboratory. H. Schlichting presents an excellent review of the conversion techniques, as applied to both smooth and rough flat plates.[6] In both cases, the basic assumptions are that the thickness

[6] "Boundary Layer Theory," by H. Schlichting, *Technical Memorandum Nos. 1217* and *1218*, National Advisory Committee for Aeronautics, Washington, D. C., 1949.

of the boundary layer corresponds to the radius of the pipe and that the shape of the velocity profile in the boundary layer is the same as that in a pipe. Although this last assumption is in error, the values of $c_f$ obtained experimentally agree in the case of the smooth plate with those predicted through Mr. Schlichting's conversion techniques.

The development of the boundary layer in nonuniform flow was studied by E. Gruschwitz, who made measurements along the converging and diverging plane walls of a channel.[7] In developing this method of analysis, Mr. Grusch-

[7] "Die turbulente Reibungschicht in ebener Strömung bei Druckabfall und Druckanstieg," by E. Gruschwitz, *Ingenieur-Archiv*, Vol. II, 1931, p. 321.

witz presents the idea that the shape of the velocity profile may be characterized by a single parameter, which is different from Λ. With this as a background, Mr. Gruschwitz numerically integrates Eq. 3 and two empirical expressions. One of these expressions (for $c_f$) is obtained by conversion of pipe-flow principles according to the method previously indicated. The other expression is for the shape parameter, which is obtained by plotting the experimentally determined values of this quantity against the pressure gradient and the Reynolds number. Mr. Gruschwitz noted little dependence on the Reynolds number, and subsequently eliminated it from consideration in the relation for the shape parameter.

Another method for the computation of boundary-layer developments in flows with pressure gradients was presented by A. E. von Doenhoff and N. Tetervin.[8] Like Mr. Gruschwitz, Messrs. Doenhoff and Tetervin consider the

[8] "Determination of General Relations for the Behavior of Turbulent Boundary Layers," by A. E. von Doenhoff and N. Tetervin, *Report No. 772*, National Advisory Committee for Aeronautics, Washington, D. C., 1943.

velocity profile susceptible to characterization by a single parameter. Unlike Mr. Gruschwitz, Messrs. Doenhoff and Tetervin chose to use Λ as this parameter. In relating Λ to the pressure gradient, they noted a consistent variation with the Reynolds number not noted by Mr. Gruschwitz. This variation was eliminated by expressing Λ as a function of the quotient of the nondimensional pressure gradient and the coefficient of the boundary shear. For the determination of this latter quantity, Messrs. Doenhoff and Tetervin used the Squire-Young formula, which was the result of experiments involving flows with both pressure increase and decrease. In this connection, Messrs. Doenhoff and Tetervin noted that "no systematic variation of the skin-friction coefficient with the shape parameter was indicated by the data."[8]

Another analysis was proposed by Messrs. Tertervin and C. C. Lin for the determination of the turbulent boundary layers with pressure gradients.[9] The

[9] "A General Integral Form of the Boundary-Layer Equation for Incompressible Flow with an Application to the Calculation of the Separation Point of Turbulent Boundary Layers," by N. Tetervin and C. C. Lin, *TN* 2158, NACA, August, 1950.

relationships obtained from laboratory observations were those between the pressure gradient and the boundary shear, as well as that between the integral of shearing stresses across the boundary layer and the pressure gradient. These relationships, together with a summary of analytical approaches to the determination of the turbulent boundary layer with pressure gradient, were obtained by P. S. Granville.[10]

[10] "A Method for the Calculation of the Turbulent Boundary Layer in a Pressure Gradient," by P. S. Granville, *Report 752*, U. S. Dept. of the Navy, The David W. Taylor Model Basin, Washington, D. C., May, 1951.

In France, G. Halbronn[11] conducted an investigation similar to that

[11] "Étude de la mise en régime des écoulements sur les ouvrages à forte pente," by G. Halbronn, *Thèse*, Université de Grenoble, Grenoble, France, 1951.

undertaken by the writer, although Mr. Halbronn made no attempt to secure a controlled variation of the slope and roughness. Mr. Halbronn used Eq. 3, although in a somewhat different form, and assumed the velocity distribution and the shear relationship to be the same as that in a pipe. There resulted an expression which enabled Mr. Halbronn to determine the boundary-layer dimensions. Treating the flow outside the boundary layer as non-viscous

flow, Mr. Halbronn was able to determine the loci of both the free surface and the upper limit of the boundary layer which was then projected to an intersection termed the critical point.

Mr. Halbronn compared the location of this critical point with observations made both in the laboratory and in the field. All Mr. Halbronn's laboratory measurements were made at values of $U\,x/\nu$ less than $4 \times 10^6$, whereas the investigations made by the writer were taken for the range from $1.4 \times 10^6$ to $3.7 \times 10^7$. Observations made by G. H. Hickox,[12] M. ASCE, of the flow on the spillway of Norris Dam at Norristown, Tenn., at values for the Reynolds numbers in the range from $1.5 \times 10^8$ to $1.5 \times 10^9$ were also compared to the analytical predictions. Further comparison was made with observations made of the flow on models of the spillway of Norris Dam and the spillway of Bort-les-Orgues Dam at Bort-les-Orgues, France. In all cases the agreement was good.

[12] "Air Entrainment on Spillway Faces," by G. H. Hickox, *Civil Engineering*, ASCE, Vol. 15, 1945, p. 562.

For a general treatment of boundary-layer characteristics the report by W. D. Baines,[13] J.M. ASCE, is more complete than that which has been presented.

[13] "A Literature Survey of Boundary-Layer Development on Smooth and Rough Surfaces at Zero Pressure Gradient," by W. D. Baines, *ONR Report*, Iowa Inst. of Hydraulic Research, Iowa City, Iowa, 1952.

## Experimental Aspects

*Equipment.*—The approach to the problem of determining the turbulent boundary was made largely from an experimental standpoint. In order to achieve a systematic variation of the quantities involved, laboratory equipment designed especially for the project was constructed. This equipment provided for the introduction of non-turbulent flow to the crest of a steep flume of controlled slope and roughness. The dimensions were limited by the quantity of space and water available, but they were regarded as adequate for the determination of information for design purposes.

The main feature of the equipment was a glass-lined flume, which was an open channel made in two sections—one 12 ft the other 4 ft long. Connecting the flume with the stilling tank were three spillway crests—one for each of the three slopes used. The function of the stilling tank was to deliver non-turbulent, potential-like flow to the spillway. This type of flow was obtained by using a large stilling tank equipped with a series of baffles, grids, and screens arranged in circular arcs. Two objectives were achieved by this arrangement: First, the presence of a pressure drop across each baffle, grid, and screen produced an effect which distributed the velocity in a potential-like pattern; second, the arrangement of coarse to fine gradation of the baffles, grids, and screen reduced the turbulence of the incoming flow to such a small scale that it was effectively dissipated before it reached the spillway crests, so that essentially the only turbulence present within the flow on the slope was that engendered by the boundary itself.

Several sizes of stagnation tubes were used to make the stagnation-pressure surveys, but after experience with each, two were picked for general use. One was relatively small, with a diameter of 0.021 in., and one relatively large, being 0.084 in. in diameter. Results of the two were compared to discover at

what distance from the boundary they began to agree. This point was reached when the center line of the large tube was approximately a diameter from the boundary. The fluctuations of the level of the mercury in the open manometer used with the large tube were evident as long as the tube was located within the boundary layer; they disappeared completely when the tube was moved to the non-turbulent zone outside the boundary layer. The line between the two domains was determinable with surprising precision on the basis of the presence and absence of the fluctuations, although the line was always outside the locus of $y = \delta$ by approximately a diameter of the tube. For the majority of the work, however, the small tube was used, since the absence of fluctuations in the open manometer permitted more precise measurement of the stagnation pressure.

To determine the precise location of the tube with respect to the boundary, an Ames dial graduated to 0.001 in. was connected to the stagnation-tube support. The origin of the $y$-coordinate was determined in two ways. The first method was visual, the tube being viewed in a direction normal to its long axis with the observer's eye slightly above the plane of the glass floor, so that both the tube and its reflection were seen simultaneously. The second involved the use of a metal shim of known thickness placed between the glass and the tube; on initial contact an electrical circuit was completed. Since the visual method could be used under water, and since it agreed with the second method to within 0.001 in., it was used almost exclusively for all work done on the glass boundary. The value of the $y$-coordinate for all runs made with the rough boundary was measured from the mid-plane of the screen to the center line of the tube.

One series of runs was made with the glass boundary and a second series of runs with a boundary roughened by stretching a length of screen over the floor of the flume. The screen was 16-by-16 mesh, bronze wire of 0.010-in. diameter. This screen was chosen because it was uniform, easily reproducible, convenient to apply and remove, and low in cost. It was important to use a piece of screen completely free of folds, bulges, and kinks; only a perfect piece would lie flat on the glass without the use of an adhesive. In order to determine the effective roughness, a piece of screen from the same roll was applied to the floor of another laboratory flume. Uniform flow was established over the screen. With knowledge of the discharge, the depth, and the slope, the values of $f$, $k$, and $n$ could be computed. The equations used to determine $f$, the coefficient of friction, and $n$, the coefficient of roughness, were the Weisbach-Darcy and Manning expressions, respectively. The equations used to determine $k$, the equivalent Nikuradse roughness height, resulted from an application of the von Kármán-Prandtl velocity-distribution equations to the case of two-dimensional flow, as given by Mr. Rouse[4]—

$$\frac{1}{\sqrt{f}} = 2.03 \log_{10} \frac{y_0}{k} + 2.11 \qquad (6)$$

—in which $y_0$ is the total thickness of the flow measured normal to the boundary. The resulting values of $k$ and $n$ indicated a roughness comparable to that of rough concrete. Thus, the development of the boundary layer over

the 16-by-16 mesh is considered to be a good approximation to that occurring over rough concrete.

*Data.*—In making the stagnation-pressure surveys the measuring carriage was located at a fixed position on the flume and a normal traverse made. After making ten to thirty stagnation-pressure measurements at different normal distances from the boundary, the carriage was moved to the next station and the process repeated. The result was a characterization of the stagnation-pressure field by knowledge of its magnitude at many discrete points. Typical velocity profiles, drawn through the experimental points, are shown in Figs. 2 and 3, for a channel slope of 40° and a discharge of 3.68 cu ft per sec per ft of channel width.

In addition to the stagnation-pressure surveys, certain other observations were made by visual inspection of the appearance of the flow. The first thing to be noted was the effect of the walls of the flume in producing small waves which issued normal to the walls. Being superimposed on the general downstream motion of the water, the waves had a total velocity which was the vector sum of the celerity and the surface velocity of the fluid over which the wave was being propagated. Since the celerity of the waves was decreasing while the celerity of the fluid was increasing, the motion of the wave took place along a curve of such a shape that a region in the form of an inverted cusp in the central portion of the flow was largely unaffected by the presence of side boundaries.

Also to be observed was the fact that the free surface in the neighborhood of the critical point was characterized by a change within a relatively short distance from a smooth, glass-like transparent appearance to one which was rough and nearly opaque. The apparent location of this transition was compared to the graphical location of the critical point. Comparison showed that slight disturbances of the surface occur upstream from the critical point because there is some turbulence outside the nominal boundary layer if the boundary-layer thickness is defined as occurring at $u/U = 0.99$. The effect intensifies rapidly from this point on, however, and reaches a well-developed stage at the critical point.

Careful measurement showed that energy loss in the domain outside the boundary layer is practically nonexistent. A flexible translucent tube completely filled with water was connected to the stagnation tube when the tube was located in the flow outside the boundary layer at the greatest possible distance from the stilling tank. The water level in this translucent tube rose to a distance above the water in the stilling tank almost exactly equal to the computed value of the velocity head at the point of comparison. If there was any energy loss, its magnitude was less than 0.001 ft in a region where the velocity head was about 10 ft.

## Analysis of the Data

*Dynamic Analysis.*—Since the von Kármán momentum equation forms a major part of the dynamic analysis of the turbulent boundary layer, it is important to review its derivation. The net change in the momentum flux as the fluid passes through an elementary volume within the boundary layer,

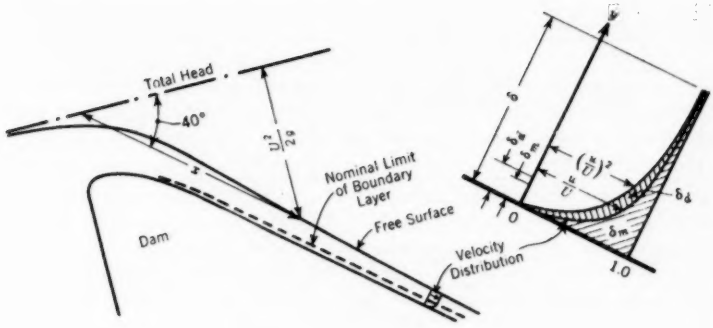


FIG. 1.--DEFINITION OF PARAMETERS

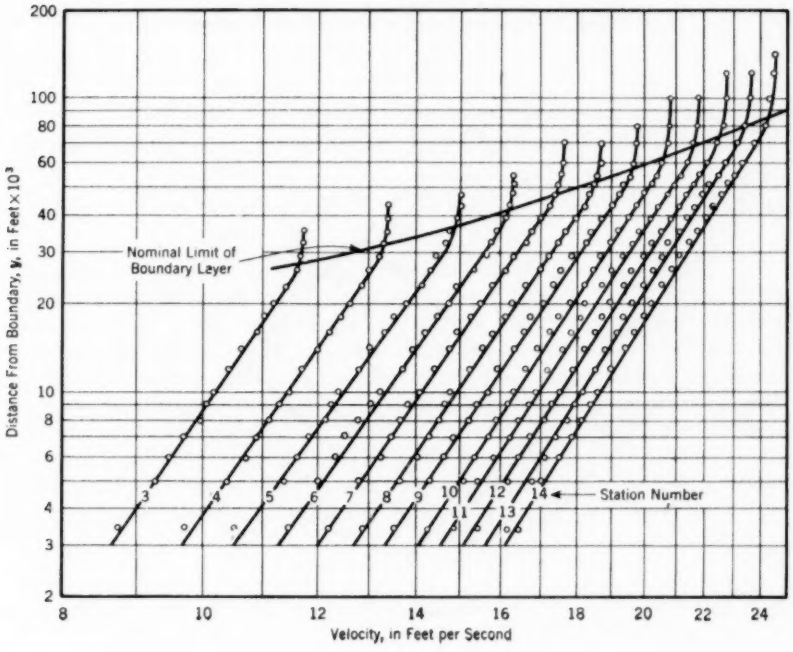


FIG. 2. - VELOCITY PROFILES WITH GLASS BOUNDARY

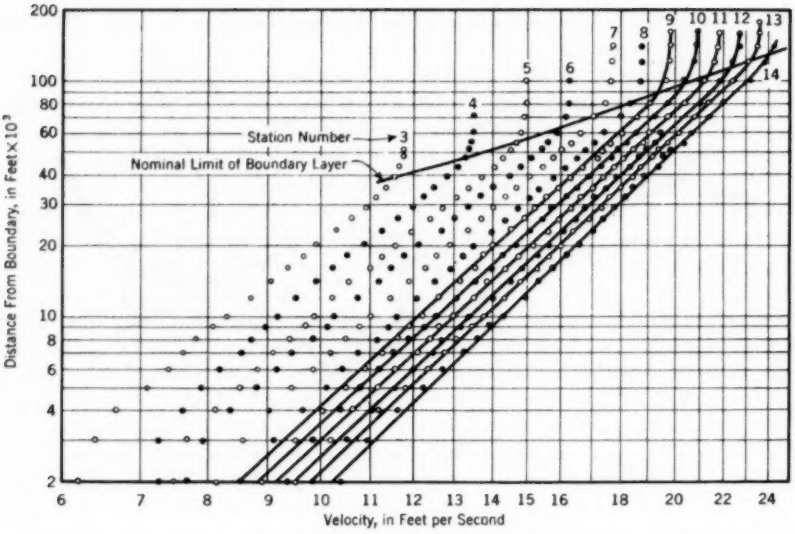


FIG. 3.—VELOCITY PROFILES WITH SCREEN BOUNDARY

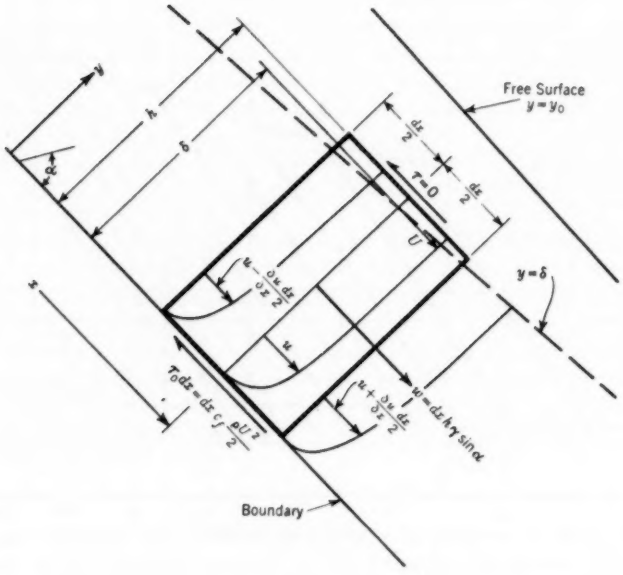


FIG. 4.—DEFINITION SKETCH

such as that shown in Fig. 4, is evaluated. The net change in the momentum flux in the $x$-direction is equated to the algebraic summation of all the forces in that direction which act on this elementary volume of fluid. The integrals which arise from this application of the principle of conservation of momentum are simplified by using the definitions of $\delta_d$ and $\delta_m$.

The forces acting on the elementary volume of fluid are the shear force at the boundary, the gravitational force, and the force due to the pressure gradient in the $x$-direction. The first of these forces can be expressed in terms of the coefficient of the local boundary shear. The combined effect of the last two forces is expressed in terms of the acceleration outside the boundary layer. This is done because this effect is not a function of distance from the boundary, and it is only the combination of the gravity and pressure gradient which produces acceleration of the fluid outside the boundary layer. On equating the net change in momentum flux to the summation of the forces acting on the elementary volume of fluid, there results

$$\rho U^2 \left[ -\frac{\partial \delta_m}{\partial x} - \frac{1}{U}\frac{\partial U}{\partial x}(2\delta_m + \delta_d) + \frac{\delta}{U}\frac{\partial U}{\partial x} \right] dx = -c_f \rho \frac{U^2}{2} dx + \rho \delta U \frac{\partial U}{\partial x} dx \quad (7)$$

From Fig. 1,

$$x = \frac{U^2}{2 g S} \quad (8)$$

in which $g$ is the acceleration of gravity and S denotes the sine function of the angle of inclination with the horizontal. Substituting this definition into Eq. 7 yields

$$c_f = 2\frac{\partial \delta_m}{\partial x} + \frac{\delta_d}{x} + 2\frac{\delta_m}{x} \quad (9)$$

which is an approximation of the von Kármán momentum equation.

Eq. 9 involves approximations because the difference between the slope of the water surface and that of the boundary is neglected. In most spillway problems, this difference is negligible in the zone of interest. Another approximation lies in the fact that the effect of the pressure gradient is neglected, and in the fact that the origin of $x$ is not fixed whereas the differentiation with respect to $x$ presumes that it is. Neither of these approximations introduces serious error, however, except in regions near the crest.

*Dimensional Analysis.*—Noting that the viscosity, density, and unit weight of the fluid enter the problem, the following relationships can be formed:

$$\delta = f_1 (U, k, x, \mu, \rho, g, q, S) \quad (10)$$

$$c_f = f_2 (U, k, x, \mu, \rho, g, q, S) \quad (11)$$

$$\Lambda = f_3 (U, k, x, \mu, \rho, g, q, S) \quad (12)$$

in which $\mu$ is the dynamic viscosity of the fluid and $q$ is the discharge per foot of width of spillway. Dimensionless combinations of these variables can be

formed:

$$\frac{\delta}{x} = \phi_1\left(\frac{U\,x}{\nu}, \frac{k}{x}, \frac{U^2}{g\,x}, \frac{q}{U\,x}, S\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$c_f = \phi_2\left(\frac{U\,x}{\nu}, \frac{k}{x}, \frac{U^2}{g\,x}, \frac{q}{U\,x}, S\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

and

$$\Lambda = \phi_3\left(\frac{U\,x}{\nu}, \frac{k}{x}, \frac{U^2}{g\,x}, \frac{q}{U\,x}, S\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

From the definition of $x$, $\frac{U^2}{g\,x}$ is directly proportional to the slope of the boundary, so that in general the three dependent variables (Eqs. 13, 14, and 15) are functions of the Reynolds number, the relative roughness, the slope, and the ratio of the potential depth to $x$, the latter ratio being an interpretation of the dimensionless expression for the discharge. In the expressions for $c_f$ and $\Lambda$, $\delta$ can be substituted for $x$ without altering the validity of the procedure. The result is that the Reynolds number and the relative roughness are written in terms of the boundary-layer thickness. This immediately furnishes a basis for comparison with uniform flow in pipes or open channels. If $x$ is retained, plots are obtained similar to those obtained from research on flat plates in uniform flow.

*Analytical Techniques.*—The values for $\delta_d$ and $\delta_m$ were obtained through numerical integration, in which the trapezoidal rule was used. An exception occurred in dealing with the region immediately adjacent to the boundary in which no stagnation-pressure measurements were possible. In this case an assumption was made which was roughly equivalent to the extrapolation of the velocity profile, according to a function of the form $u = J\,y^j$.

A graphical method was used to determine $\delta$. The straight-line portion of a log-log plot of the velocity profile was extrapolated to its intersection with the line $u = U$. The value of $y$ at this intersection was taken to be the value of $\delta$. This method depends less on visual judgment than does reading from a somewhat arbitrary curve the value of $y$ at which $u/U = 0.99$. This method also gives results that correspond roughly to the definition of $\delta$ which is associated with $u/U = 0.99$.

The magnitude of $U$ was determined by equating the vertical distance between the water surface and the line of the total head to $\frac{U^2}{2\,g}$. The slight difference between the location of the actual surface and that of the potential flow is insignificant in this computation, since the effect on $U$ is a change in magnitude of about 0.05%.

*Dimensional Plots.*—The results of the analysis of the velocity profile are presented in dimensional form to afford a graphical view of the data. In Fig. 5, $\delta$ is plotted as a function of $x$, the points resulting from tests with both glass and screen boundaries, with all three slopes, and with various rates of discharge. All the points shown in Fig. 5 are presented in dimensionless form in Fig. 6.

*Dimensionless Plots.*—Fig. 6 shows $\delta/x$ as a function of the Reynolds number; the results obtained from different slopes, from different boundaries, and from different discharges are plotted on the same system of coordinates. In addition to the writer's observations, the values obtained by Mr. Halbronn and Mr. Hickox are also plotted. Neither of these experimenters presented data in the form plotted, so that it was necessary to deduce the points from the information which was presented. In Mr. Halbronn's work, the value of $\delta$ was obtained from the velocity profile by the same method applied to the data of this investigation. In the case of the observations made by Mr. Hickox on the spillway of Norris Dam, the thickness of the boundary layer was taken to be $\delta_d$ plus the depth of the potential flow at the same discharge and at the same station on the slope.

Fig. 7 shows $\delta/x$ as a function of the relative roughness, with results obtained from different slopes and different discharges being plotted on the same system of coordinates. The observations made by Mr. Hickox are plotted, with the assumption that the $k$ of the concrete on the spillway of Norris Dam was 0.005 ft, corresponding to an average concrete surface texture. The value of $k$ for the screen was taken as 0.009 ft, according to the results of the tests in uniform flow.

Fig. 8 shows $c_f$ for both smooth and rough boundaries as a function of $\mathbf{R}$, the Reynolds number. The value of $c_f$ was computed by applying Eq. 9 to the data obtained from the smooth curves drawn in plots of $\delta_d$ and $\delta_m$ as functions of $x$. The curve in Fig. 9 has the equation,

$$\frac{1}{\sqrt{c_f}} = 4.06 \log \sqrt{c_f}\,\mathbf{R}_\delta + 3.28, \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which $\mathbf{R}_\delta$ is the Reynolds number of the fluid in the boundary layer. Eq. 16 was obtained by substituting into Eq. 4 the point $y = \delta$, $u = U$, and expressing the boundary shear in terms of $c_f$.

Fig. 9 is the counterpart of Fig. 8, and shows $c_f$ as a function of the relative roughness. A dashed line is drawn through the data and its equation is

$$\frac{1}{\sqrt{c_f}} = 4.06 \log \frac{\delta}{k} + 8.56 \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

In Eq. 17, $k$ for the screen was assumed to be 0.009 ft.

Figs. 10(*a*) and 10(*b*) show the variation of the shape parameter $\Lambda$ as a function of the Reynolds number and the relative roughness, respectively. The variation of $\Lambda$ with the Reynolds number in flow over smooth flat plates has been approximated by Messrs. Tetervin and Lin[9] by the expression:

$$\log_{10} \Lambda = 0.5990 - 0.1980 \log_{10} \mathbf{R} + 0.0189 \, (\log_{10} \mathbf{R})^2 \ldots\ldots\ldots (18)$$

in which $\mathbf{R} = U\,\delta_m/\nu$ and $\nu$ is the kinematic viscosity of the fluid. A smooth curve representing Eq. 18 and the data obtained in this investigation are shown in Fig. 10(*a*).

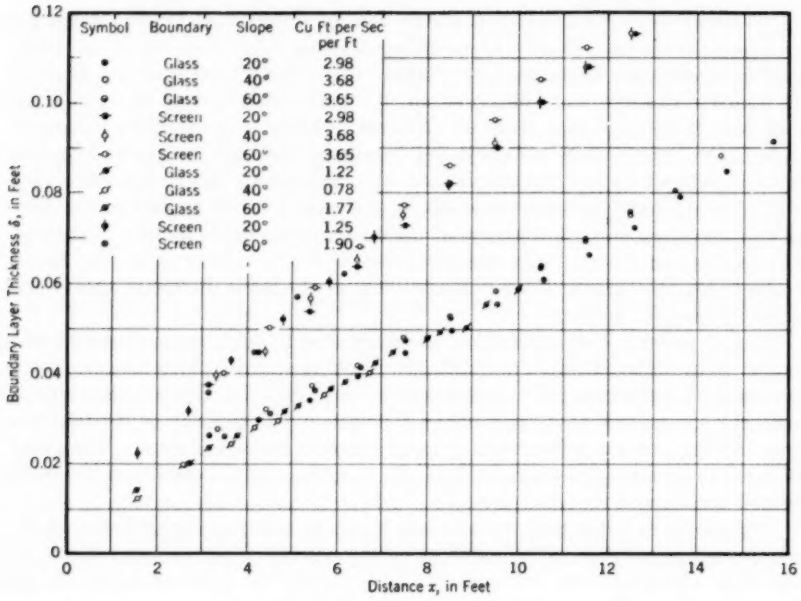


FIG. 5.—BOUNDARY-LAYER THICKNESS AS A FUNCTION OF THE DISTANCE IN THE DIRECTION OF FLOW

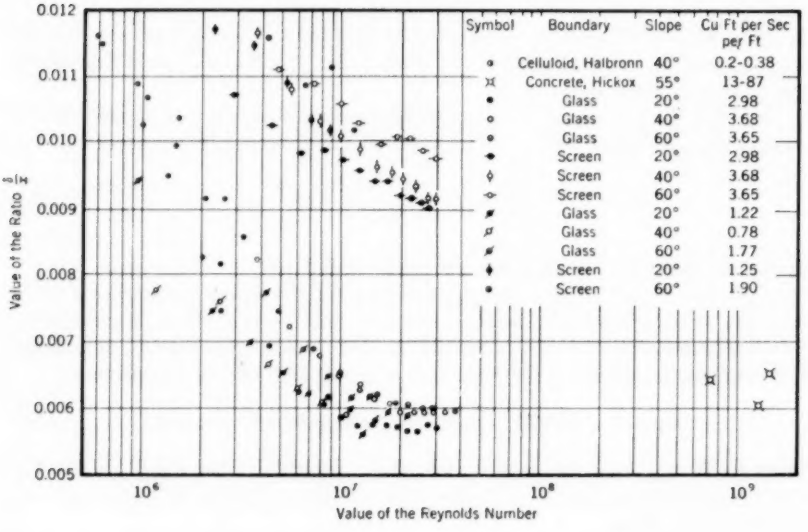


FIG. 6.—THE RATIO $\frac{\delta}{x}$ AS A FUNCTION OF THE REYNOLDS NUMBER

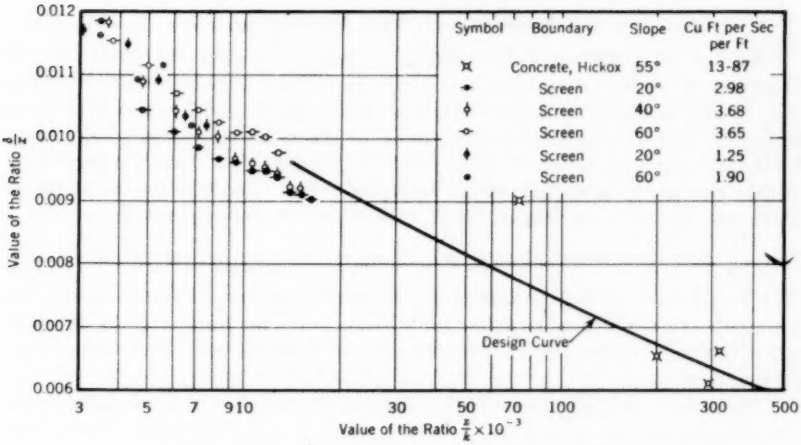
Symbol Boundary Slope Cu Ft per Sec per Ft
Concrete, Hickox 55° 13-87
Screen 20° 2.98
Screen 40° 3.68
Screen 60° 3.65
Screen 20° 1.25
Screen 60° 1.90
Design Curve
Value of the Ratio $\frac{\delta}{x}$
Value of the Ratio $\frac{x}{k} \times 10^{-3}$

FIG. 7.—THE RATIO $\frac{\delta}{x}$ AS A FUNCTION OF THE RELATIVE ROUGHNESS

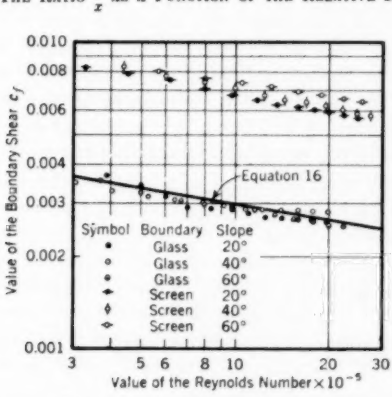


FIG. 8.—THE COEFFICIENT OF BOUNDARY SHEAR AS A FUNCTION OF THE REYNOLDS NUMBER

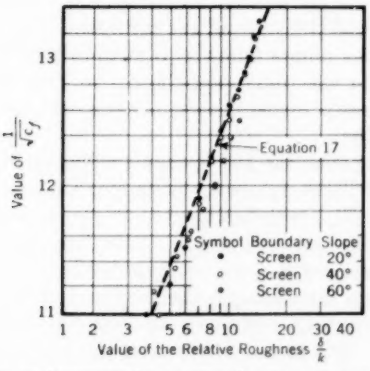


FIG. 9.—THE COEFFICIENT OF BOUNDARY SHEAR AS A FUNCTION OF THE RELATIVE ROUGHNESS

## Discussion of the Results

*Role of the Discharge.*—The discharge has an insignificant effect on the thickness of the boundary layer at a fixed value of $x$. This fact is illustrated in Fig. 5, and is shown in dimensionless form in Figs. 6 and 7. On inspection, it can be seen that points corresponding to the same slope and to the same boundary, but with much different rates of discharge, fall along the same line, within the limits of experimental variation. This does not mean the same thickness at the same station; it is only with reference to the parameter $x = \frac{U^2}{2\,g\,S}$ that the effect of the variable discharge becomes indeterminably small. This fact is basic to the solution of the full-scale problem. Therefore, only the parameter $x$ need be used to define the position along the slope, and the development becomes identical to that occurring over the same boundary at any rate of discharge.

The discharge is significant in determining the depth of the flow, and hence the location of the critical point. The presence of a free surface limits the growth of the boundary layer, so that knowledge of the discharge is necessary to define that aspect of boundary-layer development.

*Effect of the Slope.*—As in the case of the discharge, slope has little effect on the thickness of the boundary layer at a fixed value of $x$. Although a slight difference between the developments occurring at the different slopes is apparent in some instances, it will be shown in the subsequent example that no important difference in the computed location of the critical point results from choosing slightly different values of $\delta/x$. As in the case of the discharge, knowledge of the slope is essential in computing the depth of flow, and hence the location of the critical point.

The effect of the slope on $c_f$ and on $\Lambda$ is similar to its effect on $\delta/x$, with consistent variation apparent in only a few instances. In the case of the rough boundary, both $\delta/x$ and $c_f$ appear to have a positive correlation with slope. A less distinct trend in the same direction is indicated in the plot of $\Lambda$ as a function of relative roughness (Fig. 10(*b*)). The effect in the case of the smooth boundary is indefinite, however.

*Role of the Boundary Roughness.*—Of all the variables encountered, the roughness of the boundary proved to be the most significant in determining the locus of the nominal thickness of the turbulent boundary layer. In Fig. 5, it is noted that all the points follow two distinct trends—one for the smooth boundary and one for the rough boundary. In Figs. 6 and 8, this grouping according to roughness is evident, although the delineation in Fig. 6 is not so distinct as that in Fig. 5.

The difference between the two trends evident in Fig. 6 is greatest in the middle range of Reynolds numbers, being small at very low and very high Reynolds numbers. This is to be expected on two accounts: First, at very low Reynolds numbers, the flow within the boundary layer is laminar, so that roughness of the boundary is of no consequence; and second, as the Reynolds number increases with increasing $x$, the thickness of the boundary layer is so large compared to the height of the roughness $k$ that the boundary becomes effectively smooth. The latter fact is significant to the spillway problem

because it makes possible, at high Reynolds numbers, a useful computation for the location of the critical point without a precise specification for the roughness of the boundary.

The roughness of the screen used in the laboratory investigation is probably greater than that encountered on most concrete spillways. The variation of the parameter $\delta/x$ on an actual concrete spillway should lie somewhere between that associated with the glass boundary and that associated with the screen boundary. Even if the concrete were as rough as the screen, however, the effect on the location of the critical point would not be great; hence, with the possible exception of the crest region, a very smooth spillway surface is not economically justified so far as its effect upon boundary-layer development is concerned.

The slope and the rate of discharge are unimportant, therefore, as far as the rate of growth of the boundary layer is concerned. The roughness of the boundary is significant near the crest—becoming less important with increasing distance along the slope.

*Nominal Thickness of the Boundary Layer.*—At high Reynolds numbers on both smooth and rough boundaries, $\delta$ is approximately a linear function of $x$. This is especially true with the smooth boundary. It is also true that the ratio $\delta/x$ continues to decrease as the Reynolds number increases; its rate of change is small enough, however, so that very little curvature of the locus $y = \delta$ results.

The variation of $\delta_m$ can be determined from the von Kármán momentum equation as follows:

$$\frac{d\delta_m}{dx} = \frac{1}{2}\left[c_f - (\Lambda + 2)\frac{\delta_m}{x}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

The gradient of $\delta_m$ with respect to $x$ is a function of the difference between two terms, both of which are decreasing in magnitude in the $x$-direction. In the usual case, $c_f$ decreases more rapidly than does $\delta_m/x$, so that the locus of $\delta_m$ is slightly concave downward. The nature of the equation is such, however, that little curvature is possible.

A significant effect is evident in the relationship between $\Lambda$, $\delta$, and $\delta_m$. Assuming that the velocity profile may be characterized by an expression of the form $u = M\,y^m$, applying the definition of $\delta_m$, and solving for the ratio of $\delta/\delta_m$ in terms of $\Lambda$, it is found that

$$\frac{\delta}{\delta_m} = \frac{\Lambda\,(\Lambda + 1)}{\Lambda - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

The significance of this expression is that $\Lambda$ is approaching unity in the region of interest. The denominator of the fraction on the right-hand side of Eq. 14 therefore decreases more rapidly than does the numerator. This means that $\delta$ is an increasing multiple of $\delta_m$ as the shape of the velocity profile progresses in the direction of a rectangular distribution. Thus, it is necessary that the locus of $\delta$ with respect to $x$ be less concave in the direction toward the boundary than the locus of $\delta_m$ with respect to $x$. It is entirely possible, therefore, that the locus of $\delta$ with respect to $x$ is actually linear at high Reynolds numbers,

even if the locus of $\delta_m$ is not.

In the case of the rough boundary, the pronounced curvature of the locus of $\delta$ with respect to $x$ is due primarily to the continuing decrease in the relative roughness, producing a more rapid change in the value of $c_f$ as a function of $x$ than is observed in the case of the smooth boundary. As the relative roughness approaches the limit at which it is effectively smooth, however, the variation in $c_f$ will decrease, and at the same time the value of $\Lambda$ will be near unity, so that again the locus of $\delta$ with respect to $x$ is possibly linear.

*Shape of the Velocity Profile.*—The shape of the velocity profile associated with the development of the turbulent boundary layer on steep slopes is better approximated in most regions by an expression of the form $u = J\, y^i$ than one of the form $u = b \log y$. This is especially true at large values of the Reynolds number; it is not true at low values of the Reynolds number and within close proximity of the boundary. However, Eq. 18, obtained from consideration of boundary-layer developments over flat plates with zero pressure gradient, is an approximation to the values of $\Lambda$ obtained experimentally. Fig. 10 indicates the degree of agreement. The effect of continuous acceleration over the experimental range has little bearing on the value of $\Lambda$ expressed as a function of $U\, \delta_m/\nu$, as it is in Eq. 18. Hence, the value of $\Lambda$ in boundary-layer development on steep slopes does not increase subsequently according to the predictions of Eq. 18, but continues to decrease at a diminishing rate. This is based on the consideration that large values of $\Lambda$ are associated with imminent separation. It is impossible to imagine imminent separation in flow down steep slopes.

*Coefficient of Boundary Shear for the Glass.*—Application of Eq. 4 to the turbulent boundary layer on the glass steep slope yields an expression for $c_f$ which is an excellent approximation to the results obtained experimentally. This is evident upon examination of the relative position of the smooth curve and the points plotted for the glass boundary in Fig. 8. Mr. Halbronn[11] made this assumption in his method of synthesis of the development. The presence of acceleration evidently has little influence on the ratio between the intensity of boundary shear and the dynamic pressure at the edge of the boundary layer. Therefore, despite the fact that the shape of the velocity profile within the boundary layer is different from that associated with flow in a pipe, the intensity of boundary shear is the same for the same maximum velocity, the same density, and the same Reynolds number.

*Coefficient of Boundary Shear for the Screen.*—In a similar manner, application of Eq. 5 to the turbulent boundary layer on the steep slope with the screen boundary yields an expression for $c_f$ which is in excellent agreement with the experimental results, if the value of $k$ for the screen is chosen to be a great deal less than the value indicated by a test of the screen in uniform flow in an open channel. The value of $k$ which gives an expression equivalent to one derivable from Eq. 5 is 0.0021 ft; the value indicated by the test in uniform flow was 0.009 ft. The value of 0.0021 ft is nearly the thickness of the screen, two wire diameters being 0.0017 ft. However, both values of $k$ indicate a roughness comparable to concrete with corresponding values of $n$ equal to 0.012 and 0.015, respectively.

Any derivation for $c_f$ as a function of the relative roughness which assumes the boundary shear to be composed entirely of form drag on elements of roughness, and which approximates the velocity profile by an expression of the form $u = b \log y$, will yield an expression of the form

$$\frac{1}{\sqrt{c_f}} = M \log \frac{\delta}{k} + B \qquad (21)$$

in which the ratio $M/B = \log 30. = 1.477$. However, if it is assumed that the distribution of the velocity if of the form $u = J\, y^j$, there results

$$\log \frac{1}{\sqrt{c_f}} = j \log \frac{\delta}{k} + C_{v,r} \qquad (22)$$

in which $C_{v,r}$ is a constant dependent on the shape of the velocity profile and the height, shape, and density of the roughness.

The actual value of the roughness of the screen as expressed by a magnitude for $k$ remains in doubt over a range of values which are associated with various concrete surface textures. The fact that the value lies within this range, however, renders the observations made in flow over the screen useful.

## Application to the Design Problem

*Outline of the Method of Approach.*—A design problem requires the location of the critical point for a given discharge on a given spillway and the computation of the water-surface profile upstream from that point. The location of the critical point is determined by the graphical intersection of the projection of the locus of the nominal thickness of the turbulent boundary layer with the projection of the locus of the free surface. The locus of the nominal thickness of the boundary layer is determined with reference to either Fig. 6 or Fig. 7, depending on the roughness of the surface. The locus of the free surface is determined by adding to the depth of the potential flow the displacement thickness of the boundary layer, which is assumed to be from $\frac{1}{8}$ to $\frac{1}{10}$ of the thickness of the boundary layer, depending on the magnitude of the Reynolds number.

*Illustrative Example.*—A concrete spillway of indefinite length, with S equal to 0.80, and a roughness such that $k$ is 0.005 ft., requires location of the critical point and the line of the free surface when the discharge is 360 cu ft per sec per ft of the spillway width.

With reference to the "Design Curve" of Fig. 7, computations can be made in the form shown in Table 1. The distance $x$ is taken to be any reasonable

TABLE 1.—Solution of the Illustrative Problem

| $x$, in feet | $\frac{x}{k}$ | $\frac{\delta}{x}$ | $\delta$, in feet | $\frac{U^2}{2g}$, in feet | $\frac{q}{U}$, in feet | $y_0$, in feet |
|---|---|---|---|---|---|---|
| 50 | $1\times10^4$ | 0.0073 | 0.36 | 40 | 7.09 | 7.13 |
| 100 | $2\times10^4$ | 0.0066 | 0.66 | 80 | 5.00 | 5.07 |
| 200 | $4\times10^4$ | 0.0060 | 1.20 | 160 | 3.53 | 3.65 |
| 400 | $8\times10^4$ | 0.0055 | 2.20 | 320 | 2.50 | 2.72 |
| 600 | $1.2\times10^5$ | 0.0052 | 3.12 | 480 | 2.04 | 2.35 |

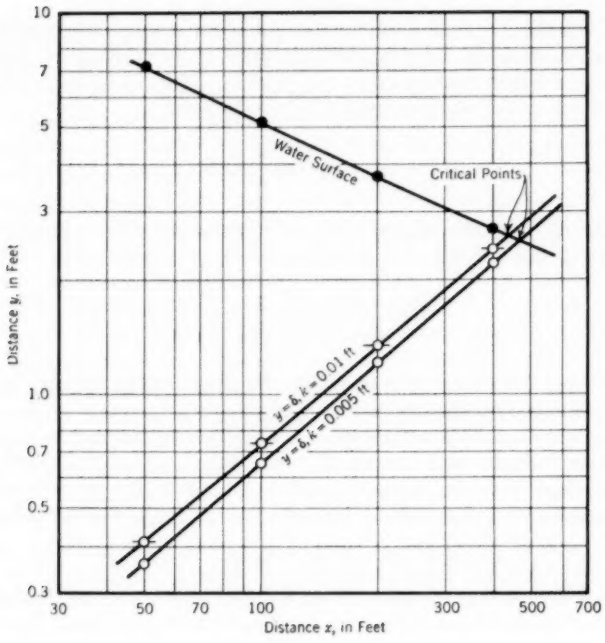


FIG. 10.—THE SHAPE PARAMETER AS A FUNCTION OF THE REYNOLDS NUMBER AND RELATIVE ROUGHNESS

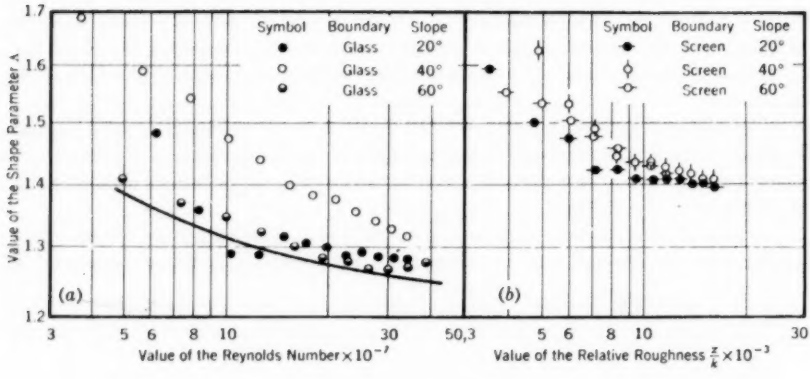


FIG. 11.—GRAPHICAL SOLUTION OF THE ILLUSTRATIVE PROBLEM

distance along the slope at which the location of the water surface is desired. The value of $x/k$ is the value of $x$ divided by 0.005 ft. The value of $\delta/x$ is read from the design curve of Fig. 7. The value of $\delta$ is obtained by multiplication of the values of $x$ and $\delta/x$. The value of $\frac{U^2}{2g}$ is by definition the product of S and $x$—in this case 0.80 $x$. The value of $U$ from the known magnitude of the velocity head having been determined, the discharge per foot is divided by $U$ to obtain the potential thickness of flow, $q/U$. To this is added 10% of the thickness of the boundary layer in order to determine the actual thickness of the flow, $y_0$.

To locate the critical point, values of $\delta$ and $y_0$ obtained from Table 1, as functions of $x$, are plotted. This plot is shown in Fig. 11, which also shows the loci of the nominal thickness of the boundary layer and of the free surface. The desired value of $x$ is found to be 470 ft.

To illustrate the effect of change in the roughness, the computation was repeated using the same situation, but with a roughness twice as large. The line "$y = \delta$, $k = 0.01$ ft" is the result of this computation. It can be seen that the difference between the values of $x$ for the two cases is about 30 ft in 450 ft—a variation of less than 10%.

The order of magnitude of the changes in $\delta/x$ produced by a difference in slope is less than that involved in changing the roughness twofold. The design curve of Fig. 7 will thus suffice for all slopes.

After the value of $x$ at the critical point has been determined, the corresponding station on the slope may be located by measuring the value of $x$ in feet along the slope. The origin of coordinates is the intersection of the line of total head with a line drawn parallel to the slope through the water surface at the critical point. This procedure satisfies the definition of $x$.

## Conclusions

The following conclusions seem justified on the basis of the foregoing results and discussion:

1. Aside from their respective influences on the location of the free surface and the magnitude of the potential velocity, neither the slope of the boundary nor the magnitude of the discharge is significant in determining $\delta$ as a function of the Reynolds number or as a function of $x/k$. The magnitude of the boundary roughness is the most significant variable in this respect, its influence decreasing with increasing boundary-layer thickness.

2. Application of the von Kármán-Prandtl velocity-distribution equations for turbulent flow near smooth and near rough boundaries to the turbulent boundary layer as it develops on both smooth and rough steep slopes yield expressions for $c_f$ which are excellent approximations of the results obtained experimentally. In the case of the rough boundary, however, such agreement requires the use of a value of $k$ which is considerably less than that determined by testing the same roughness in uniform flow in an open channel.

3. The shape of the velocity profile as it develops on smooth, steep slopes is little different from that observed in flow past flat plates. For both smooth

and rough boundaries an expression of the form $u = J\,y^j$ is a better characterization of the velocity distribution than one of the form, $u = b \log y$. In the case of the rough boundary, the value of $j$ is larger than in the case of the smooth boundary.

4. At large distances along a steep slope, the quantity $d\delta/dx$ is nearly constant and differs little for the various roughnesses encountered on concrete spillways. For this reason, a useful determination of the location of the critical point may be made without a precise specification for the value of $k$.


### Acknowledgments

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## APPENDIX. NOTATION

The following letter symbols, adopted for use in this paper and its discussion, conform essentially with American Standard Letter Symbols for Hydraulics (ASA-Z10.2-1942), prepared by a committee of the American Standards Association, with ASCE participation, and approved by the Association in 1942.

$B$ = a constant, such that $M/B = \log 30$;
$C_{v,r}$ = a constant, dependent on velocity and roughness;
$c_f$ = local coefficient of boundary shear;
$f$ = Weisbach-Darcy coefficient of friction;
$g$ = acceleration of gravity;
$J, j$ = constants in the expression $u = J\,y^j$;
$k$ = equivalent Nikuradse roughness height;
$M$ = a constant, such that $M/B = \log 30$;
$n$ = the Manning coefficient of roughness;
$q$ = discharge per foot width of spillway;
$\mathbf{R}$ = Reynolds number of the fluid;
$\mathbf{R}_\delta$ = Reynolds number of the fluid in the boundary layer;
$S$ = the slope measured as the sine function of the angle of inclination with the horizontal;
$U$ = velocity of the potential flow;
$u$ = velocity at a point within boundary layer;
$W$ = weight of an elementary volume of the fluid;

$x$ = distance in the direction of flow;

$y$ = normal distance from the boundary;

$y_0$ = total thickness of the flow measured normal to the boundary;

$\gamma$ = specific weight of the fluid;

$\delta$ = nominal thickness of the boundary layer;

$\delta_d$ = the distance that the potential flow has been displaced by the boundary layer;

$\delta_m$ = momentum thickness of the boundary layer;

$\Lambda$ = shape parameter of the velocity profile;

$\mu$ = dynamic viscosity of the fluid;

$\nu$ = kinematic viscosity of the fluid;

$\rho$ = mass density of the fluid; and

$\tau_0$ = intensity of boundary shear.

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